

Lepton Family Symmetry and Neutrino Mass Matrix

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Abstract

The standard model of leptons is extended to accommodate a discrete $Z_3 \times Z_2$ family symmetry. After rotating the charged-lepton mass matrix to its diagonal form, the neutrino mass matrix reveals itself as very suitable for explaining atmospheric and solar neutrino oscillation data. A generic requirement of this approach is the appearance of three Higgs doublets at the electroweak scale, with observable flavor violating decays.

In the standard model of particle interactions, the fermion mass matrices are *a priori* completely arbitrary, and yet data seem to indicate specific patterns which are not yet understood theoretically. Recent experimental advances in measuring the neutrino-oscillation parameters in atmospheric [1] and solar [2] data have now fixed the 3×3 lepton mixing matrix to a large extent. This information is not sufficient to fix all the elements of the neutrino mass matrix \mathcal{M}_ν (assumed here to be Majorana at the outset), but is enough to fix its approximate form in terms of a small number of parameters [3]. However, this works only in the basis $(\nu_e, \nu_\mu, \nu_\tau)$, i.e. where the charged-lepton mass matrix \mathcal{M}_l has been assumed diagonal.

If a symmetry is behind the observed pattern of \mathcal{M}_ν , then it must also apply to \mathcal{M}_l . The realization of this in a complete theory of leptons has only been done in a few cases. Some recent examples are those based on the symmetries of geometric objects, i.e. S_3 (equilateral triangle) [4], D_4 (square) [5], and A_4 (tetrahedron) [6, 7]. In this paper, a much simpler and more flexible model is proposed, based on the discrete symmetry $Z_3 \times Z_2$. [Because of the choice of representations, this turns out to be equivalent to a specific realization of S_3 .]

The key idea which allows \mathcal{M}_ν to be more restricted than \mathcal{M}_l is that they come from different mechanisms. Whereas the former comes from the naturally small vacuum expectation value (vev) of a single heavy Higgs triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$ [8], thus dispensing with the usual three heavy singlet neutrinos, the latter comes from the vev 's of three Higgs doublets [9, 10]. As shown below, the use of $Z_3 \times Z_2$ as a lepton family symmetry results in 2 parameters for \mathcal{M}_ν and 5 parameters for $\mathcal{M}_l \mathcal{M}_l^\dagger$. After rotating $\mathcal{M}_l \mathcal{M}_l^\dagger$ to its diagonal form, i.e. $(m_e^2, m_\mu^2, m_\tau^2)$, then under a condition to be derived below, the neutrino mass matrix will become [10]

$$\mathcal{M}_\nu = \begin{pmatrix} A - B & 0 & 0 \\ 0 & A & -B \\ 0 & -B & A \end{pmatrix}, \quad (1)$$

which is very suitable as a starting point for explaining atmospheric and solar neutrino oscillations. The addition of a charged Higgs singlet χ^+ and the soft breaking of $Z_3 \times Z_2$ will then lead to a complete satisfactory description of all data, including the possibility of CP violation, i.e a nonzero complex U_{e3} .

The representations of Z_3 are denoted by 1, ω , ω^2 , where

$$\omega = e^{2\pi i/3} = -\frac{1}{2} + i\sqrt{\frac{3}{2}}, \quad (2)$$

with $1 + \omega + \omega^2 = 0$. Let the standard model be augmented with a Higgs triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$, three doublets $\phi_i = (\phi_i^0, \phi_i^-)$ and a charged singlet χ^+ , in addition to the usual three lepton doublets (ν_i, l_i) and three singlets l_i^c . Under Z_3 , let

$$l_1, l_1^c, \phi_1, \xi, \chi \sim 1, \quad (3)$$

$$l_2, l_2^c, \phi_2 \sim \omega, \quad (4)$$

$$l_3, l_3^c, \phi_3 \sim \omega^2. \quad (5)$$

Under Z_2 , let

$$l_2 \leftrightarrow l_3, \quad l_2^c \leftrightarrow l_3^c, \quad \phi_2 \leftrightarrow \phi_3, \quad \chi \leftrightarrow -\chi. \quad (6)$$

Then the Yukawa couplings of

$$\nu_i \nu_j \xi^0 - \left(\frac{\nu_i l_j + l_i \nu_j}{\sqrt{2}} \right) \xi^+ + l_i l_j \xi^{++} \quad (7)$$

imply that \mathcal{M}_ν is of the form [11]

$$\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}. \quad (8)$$

Note that \mathcal{M}_ν is proportional to $\langle \xi^0 \rangle$ which can be naturally small if m_ξ^2 is positive and large. On the other hand, the Yukawa couplings of

$$l_i^c (l_j \phi_k^0 - \nu_j \phi_k^-) \quad (9)$$

imply that the mass matrix linking l to l^c is given by

$$\mathcal{M}_l = \begin{pmatrix} c & f & f \\ g & d & e \\ g & e & d \end{pmatrix}, \quad (10)$$

where $v_2 = v_3$ has been assumed for $\langle \phi_k^0 \rangle$. [This assumption will be relaxed later to accommodate the soft breaking of Z_2 . The role of χ^+ will also be explained.]

To rotate \mathcal{M}_l to its diagonal form, i.e.

$$V_L \mathcal{M}_l V_R^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (11)$$

consider

$$\mathcal{M}_l \mathcal{M}_l^\dagger = \begin{pmatrix} C & F & F \\ F^* & D & E \\ F^* & E & D \end{pmatrix}, \quad (12)$$

where

$$C = |c|^2 + 2|f|^2, \quad (13)$$

$$D = |d|^2 + |e|^2 + |g|^2, \quad (14)$$

$$E = de^* + ed^* + |g|^2, \quad (15)$$

$$F = cg^* + f(d^* + e^*). \quad (16)$$

Then

$$V_L \mathcal{M}_l \mathcal{M}_l^\dagger V_L^\dagger = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} \quad (17)$$

and the neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau)$ is given by

$$\mathcal{M}_\nu = V_L \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix} V_L^T. \quad (18)$$

As a trial, let

$$V_L = \begin{pmatrix} 0 & -i/\sqrt{2} & i/\sqrt{2} \\ -1/\sqrt{2} & 1/2 & 1/2 \\ 1/\sqrt{2} & 1/2 & 1/2 \end{pmatrix}, \quad (19)$$

then $V_L \mathcal{M}_l \mathcal{M}_l^\dagger V_L^\dagger$ becomes

$$\begin{pmatrix} D - E & 0 & 0 \\ 0 & (C + D + E)/2 - \sqrt{2} \text{Re} F & (D + E - C)/2 - i\sqrt{2} \text{Im} F \\ 0 & (D + E - C)/2 + i\sqrt{2} \text{Im} F & (C + D + E)/2 + \sqrt{2} \text{Re} F \end{pmatrix}. \quad (20)$$

Comparing this with Eq. (17), it is clear that under the condition

$$C = D + E, \quad \text{Im} F = 0, \quad (21)$$

the charged-lepton matrix is diagonalized by the V_L of Eq. (19) with

$$m_e^2 = D - E, \quad (22)$$

$$m_\mu^2 = (C + D + E)/2 - \sqrt{2} \text{Re} F, \quad (23)$$

$$m_\tau^2 = (C + D + E)/2 + \sqrt{2} \text{Re} F, \quad (24)$$

and in this basis, the neutrino mass matrix of Eq. (18) is given by

$$\mathcal{M}_\nu = \begin{pmatrix} b & 0 & 0 \\ 0 & (b + a)/2 & (b - a)/2 \\ 0 & (b - a)/2 & (b + a)/2 \end{pmatrix}, \quad (25)$$

which is identical to Eq. (1) with the substitution $b = A - B$ and $a = A + B$. The eigenvalues and eigenvectors of \mathcal{M}_ν are then

$$b : \nu_e, \quad (26)$$

$$b : (\nu_\mu + \nu_\tau)/\sqrt{2}, \quad (27)$$

$$a : (-\nu_\mu + \nu_\tau)/\sqrt{2}. \quad (28)$$

This explains atmospheric neutrino oscillations with

$$\Delta m_{atm}^2 = a^2 - b^2, \quad \sin^2 2\theta_{atm} = 1, \quad (29)$$

but $\Delta m_{sol}^2 = 0$, which is nevertheless a good first approximation. To split the two degenerate neutrino masses responsible for solar neutrino oscillations, consider the Yukawa coupling

$$(\nu_2 l_3 - l_2 \nu_3) \chi^+ = -i[\nu_e(\mu + \tau)/\sqrt{2} - e(\nu_\mu + \nu_\tau)/\sqrt{2}], \quad (30)$$

which is the only one allowed under $Z_3 \times Z_2$. In addition, the most general trilinear scalar couplings

$$\chi^+(\phi_i^0\phi_j^- - \phi_i^-\phi_j^0) \quad (31)$$

are assumed, thus breaking $Z_3 \times Z_2$ but only softly. As a result, there are one-loop contributions to \mathcal{M}_ν as shown in Fig. 1, and three nonzero parameters will emerge, i.e. Δm_{sol}^2 , $\tan^2 \theta_{sol}$, and U_{e3} .

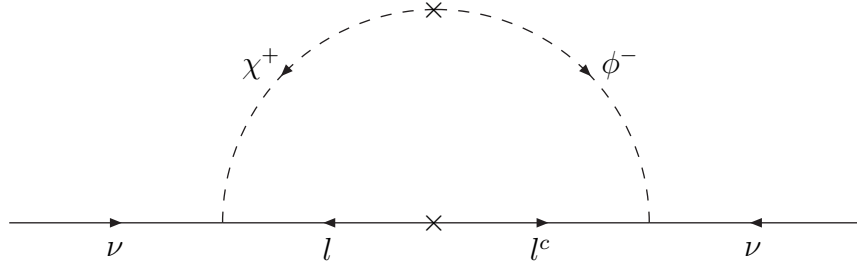


Figure 1: One-loop contributions to the neutrino mass matrix.

In the original $\nu_{1,2,3}$ basis, i.e. that of Eq. (8), the radiative corrections of Fig. 1 have the form

$$\mathcal{M}_{rad} = \begin{pmatrix} 0 & r & s \\ r & p & t \\ s & t & q \end{pmatrix}. \quad (32)$$

Rotating to the basis $\{\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}, (-\nu_\mu + \nu_\tau)/\sqrt{2}\}$ and using Eqs. (26) to (28), the neutrino mass matrix is then given by

$$\mathcal{M}_\nu = \begin{pmatrix} b+t-(p+q)/2 & i(q-p)/2 & i(s-r)/\sqrt{2} \\ i(q-p)/2 & b+t+(p+q)/2 & (r+s)/\sqrt{2} \\ i(s-r)/\sqrt{2} & (r+s)/\sqrt{2} & a \end{pmatrix}. \quad (33)$$

The two-fold degeneracy of the solar neutrino doublet is now lifted with

$$\Delta m_{sol}^2 = 4(b+t)\sqrt{pq}, \quad (34)$$

and

$$\tan^2 \theta_{sol} = \left(\frac{1-x}{1+x} \right)^2, \quad x = \sqrt{\frac{q}{p}}. \quad (35)$$

For $x \simeq 0.22$, $\tan^2 \theta_{sol} \simeq 0.41$, as indicated [12] by most recent data. A nonzero

$$U_{e3} \simeq \frac{i(r-s)}{\sqrt{2}(a-b-t)} \quad (36)$$

is also obtained. Note that the phase of U_{e3} is undetermined because $r-s$ is in general complex. The soft terms of Eq. (31) break Z_2 , hence the condition $v_2 = v_3$ assumed previously should also be relaxed, in which case the zero entries of Eq. (20) would become nonzero. This means another contribution to U_{e3} . The condition of Eq. (21) may also be relaxed to account for any possible deviation from maximum mixing in atmospheric neutrino data.

The representation content of lepton families and Higgs multiplets under $Z_3 \times Z_2$ proposed here is such that the resulting model is equivalent to the following specific realization of S_3 . There are 3 irreducible representations of S_3 : $\underline{1}^+$, $\underline{1}^-$, $\underline{2}$. If $\underline{1}^+$ and $\underline{2}$ are chosen as representations here with $\underline{1}^+$ as 1 and $\underline{2}$ as (ω, ω^2) under Z_3 , together with Z_2 of Eq. (6), then the group multiplication rules of the two are the same. For example, the S_3 invariant $\underline{2} \times \underline{2} \times \underline{2} \rightarrow 1$ is given by $(1, 2) \times (1, 2) \times (1, 2) = 111 + 222$, whereas the $Z_3 \times Z_2$ analog is $(\omega, \omega^2) \times (\omega, \omega^2) \times (\omega, \omega^2) = \omega^3 + \omega^6$. It can also be checked easily that \mathcal{M}_ν of Eq. (8) has 2 invariants and \mathcal{M}_l of Eq. (10) has 5 invariants in both cases as expected.

Diagonalizing Eq. (12) with V_L of Eq. (19) under the condition of Eq. (21) leads to the neutrino mass matrix of Eq. (25) which yields the eigenvalues and eigenvectors of Eqs. (26) to (28). Adding the one-loop radiative corrections induced by the interactions of Eqs. (30) and (31), the full \mathcal{M}_ν of Eq. (33) is then obtained. It is very suitable for explaining present data on atmospheric and solar neutrino oscillations, with $\sin^2 2\theta_{atm} \simeq 1$, $\tan^2 \theta_{sol} \simeq 0.4$, and U_{e3} small but nonzero. It also has the flexibility to allow for either the normal hierarchy $m_1 < m_2 < m_3$ or the inverted hierarchy $m_3 < m_1 < m_2$ of neutrino masses.

In this model, lepton masses come from three different sources in the Higgs sector: charged-lepton Dirac masses come from three Higgs doublets, neutrino Majorana masses come from a Higgs triplet at tree level, and the interactions of a charged Higgs singlet with the already present Higgs doublets at the one-loop level. Singlet (right-handed) neutrinos are not needed.

To have a naturally small $\langle \xi^0 \rangle$, m_ξ^2 should be positive and large [8], which means that the interactions of ξ are not observable at the electroweak scale. On the other hand, the three Higgs doublets (ϕ_i^0, ϕ_i^-) and the one Higgs singlet χ^+ should have masses below the TeV scale, resulting in observable phenomena at future colliders. The typical decay of any one of the three physical charged scalars is into a charged lepton and a neutrino, with large violations of lepton family universality. There are also five neutral scalars, each decaying into a pair of charged leptons $l_i^- l_j^+$. Since $i \neq j$ is allowed, there should be many distinct observable experimental signatures.

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